Three-Loop Corrections to the Higgs Boson Mass and Implications for Supersymmetry at the LHC

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(Received 8 July 2013; published 25 September 2013)

In supersymmetric models with minimal particle content and without left-right squark mixing, the conventional wisdom is that the 125.6 GeV Higgs boson mass implies top squark masses of $\mathcal{O}(10)$ TeV, far beyond the reach of colliders. This conclusion is subject to significant theoretical uncertainties, however, and we provide evidence that it may be far too pessimistic. We evaluate the Higgs boson mass, including the dominant three-loop terms at $\mathcal{O}(\alpha_t \alpha_s^2)$, in currently viable models. For multi-TeV top squarks, the three-loop corrections can increase the Higgs boson mass by as much as 3 GeV and lower the required top-squark masses to 3–4 TeV, greatly improving prospects for supersymmetry discovery at the upcoming run of the LHC and its high-luminosity upgrade.

DOI: 10.1103/PhysRevLett.111.131802 PACS numbers: 12.60.Jv, 12.15.Lk, 13.85.Rm, 14.80.Da

Introduction.—The Higgs boson, recently discovered at the LHC by the ATLAS and CMS Collaborations [1,2], is now the subject of impressive precision studies. In particular, combining the results of all channels, the currently available data, consisting of 25 fb^{-1} collected at $\sqrt{s} = 7$ and 8 TeV, constrain the Higgs boson mass to be [3,4]

$$\text{ATLAS(combined): } 125.5 \pm 0.2^{+0.5}_{-0.6}\ \text{GeV}, \qquad (1)$$

$$\text{CMS(combined): } 125.7 \pm 0.3 \pm 0.3\ \text{GeV}, \qquad (2)$$

where the first uncertainties are statistical and the second systematic. Because the Higgs boson has been seen in purely leptonic and photonic channels without missing E_T, its mass is already known with a fractional uncertainty smaller than any of the quarks, providing a potentially stringent bound on ideas for new physics.

The Higgs boson mass measurement is especially important for supersymmetry. In supersymmetry, the Higgs quartic coupling is determined, at tree level, by the gauge couplings, removing this *a priori* free standard model parameter. The Higgs mass m_h also receives large radiative corrections, which are functions of superpartner masses. As a result, m_h provides useful guidance as to the mass scale of the superpartners, with implications for direct discovery prospects for supersymmetry at colliders. Unfortunately, this potential is currently clouded by theoretical uncertainties in the Higgs boson mass calculation, which are arguably much larger than the experimental uncertainties. In this Letter, we extend previous work by including the dominant three-loop contributions to m_h derived in Refs. [5,6], and we explore implications for supersymmetry discovery prospects at the LHC.

The Higgs mass at three loops.—In supersymmetric models with minimal field content, the tree-level Higgs boson mass cannot exceed $m_Z \simeq 91$ GeV. The one-loop contributions were explored long ago [7–9], and many studies now incorporate two-loop contributions, available with public codes such as FEYNHIGGS [10–13], SOFTSUSY [14], SUSPECT [15], and SPHENO [16,17].

The radiative corrections to the Higgs boson mass are most sensitive to the top squark sector. At tree-level, the top squark mass matrix is

$$(\tilde{t}_L^*, \tilde{t}_R^*)\begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + \Delta_L & m_t X_t \\ m_t X_t & m_{\tilde{t}_R}^2 + m_t^2 + \Delta_R \end{pmatrix}\begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \qquad (3)$$

where $X_t \equiv A_t - \mu \cot\beta$, $\Delta_L \equiv (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W) m_Z^2 \cos 2\beta$, and $\Delta_R \equiv \frac{2}{3}\sin^2\theta_W m_Z^2 \cos 2\beta$. Diagonalizing this matrix gives the physical masses of the lighter top squark $\tilde{t}_1$ and heavier top squark $\tilde{t}_2$. The radiative contributions are maximized for heavy top squarks and large left-right mixing with $X_t/M_S \approx \sqrt{6}$, where $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. This "maximal mixing" relation is valid at one loop; it is modified by higher-order corrections, but remains within $\sim$20% of the one-loop value. For $X_t \ll M_S$, however, conventional two-loop analyses imply that the measured Higgs mass requires top squarks with masses $\sim$5–10 TeV. If this is the characteristic mass scale of all squarks, they will be far beyond the reach of the LHC or any near-future collider.

To improve the accuracy of current estimates of m_h, we use, here, the program H3M [5]. Building on the 1- and two-loop terms provided by FEYNHIGGS [10–13], H3M includes the roughly 16 000 diagrams that are the leading three-loop corrections at $\mathcal{O}(\alpha_t \alpha_s^2)$ [5,6].

When evaluating m_h, special care has to be taken to use accurate numbers for the values of the input parameters entering the calculation, most notably the top quark mass m_t and the strong coupling constant α_s in supersymmetric (SUSY) QCD, renormalized in the $\overline{\text{DR}}$ scheme [i.e., using dimensional reduction and modified minimal subtraction (MS)], at a specific renormalization scale μ. These must be calculated from the experimentally accessible values of the top quark pole mass and $\alpha_s(m_Z)$ in five-flavor QCD.

In the original version of H3M, the transition of m_t from the on-shell to the $\overline{\text{DR}}$ scheme could suffer from large logarithms if superpartners masses or renormalization scales μ are much larger than m_t. Since null results from the LHC increasingly favor this possibility, the program has been improved in the following way. First, we calculate $m_t(\mu)$ in five-flavor QCD in the $\overline{\text{MS}}$ scheme using four-loop running as implemented in the numerical package RUNDEC [18]. This value is transferred to the $\overline{\text{DR}}$ scheme via a finite renormalization at three-loop order [19,20]. Finally, the transition from five-flavor QCD to SUSY QCD is performed using the two-loop decoupling coefficient of m_t [21,22]. This procedure is faster, more robust, and more accurate than the old code [23].

Results as a function of weak-scale parameters.—We now present results for the Higgs boson mass, including the three-loop corrections described above, as functions of weak-scale supersymmetry parameters. We set $\tan\beta = 20$ so that the tree-level Higgs boson mass is within 1 GeV of its maximal value, and we consider nearly degenerate, unmixed top squarks, with $m_{\tilde{t}_L} = m_{\tilde{t}_R}$ and $X_t = 0$. The dependence on other parameters is relatively mild; we set $\mu = 200$ GeV, assume gaugino mass unification with $m_{\tilde{g}} = 1.5$ TeV, and set all other sfermion soft mass parameters equal to $m_{\tilde{t}_{L,R}} + 1$ TeV. For multi-TeV values of the sfermion masses, these models have scalar masses far heavier than gaugino and Higgsino masses.

The results are shown in Fig. 1. For $m_{\tilde{t}_1}$ in the range 1–10 TeV, one-loop corrections raise the Higgs mass by 18–31 GeV, and two-loop corrections raise the mass further by another 4 to 7 GeV. The experimental value of m_h is apparently obtained for $m_{\tilde{t}_1} \sim 5$ TeV. However, the three-loop effects raise the Higgs mass by another 0.5–3 GeV. The magnitude of the corrections decreases with increasing loop order, indicating a well-behaved, if slowly converging, perturbative expansion, and the size of the three-loop corrections is consistent, within uncertainties, with the next-to-leading logarithm analysis of Ref. [24]. Clearly, however, the three-loop corrections are still sizable, and they reduce the required top squark mass to 3–4 TeV, a reduction with potentially great significance for supersymmetry discovery, as we discuss below.

Reference [24] observes partial cancellations between leading logarithm terms of $\mathcal{O}(\alpha_t \alpha_s^2)$ and $\mathcal{O}(\alpha_t^2 \alpha_s)$ in a particular scenario. We advocate a full calculation at $\mathcal{O}(\alpha_t^2 \alpha_s)$ to investigate whether this behavior is universal.

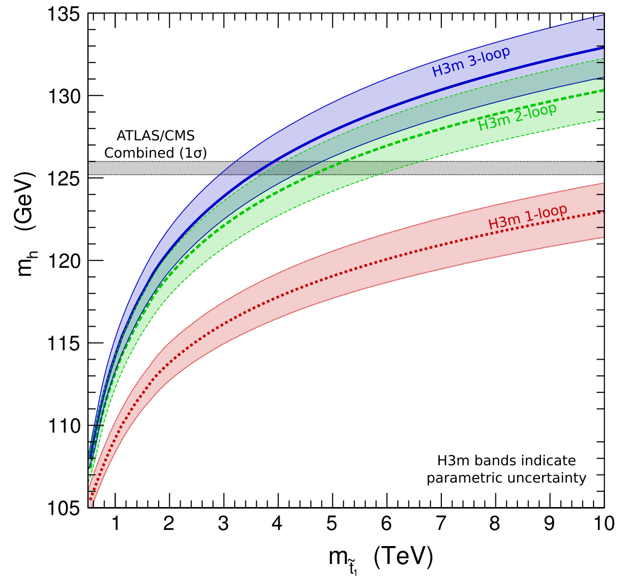

FIG. 1 (color online). The Higgs boson mass m_h from H3M at one, two, and three loops for nearly degenerate ($m_{\tilde{t}_L} = m_{\tilde{t}_R}$), unmixed ($X_t = 0$) top squarks, as a function of the physical mass $m_{\tilde{t}_1}$. The renormalization scale is fixed to $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, we set $\tan\beta = 20$, $\mu = 200$ GeV, all other sfermion soft parameters equal to $m_{\tilde{t}_{L,R}} + 1$ TeV, and assume gaugino mass unification with $m_{\tilde{g}} = 1.5$ TeV. The bands indicate the parametric uncertainty from $m_t^{\text{pole}} = 173.3 \pm 1.8$ GeV and $\alpha_s(m_Z) = 0.1184 \pm 0.0007$. The horizontal bar is the experimentally allowed range $m_h = 125.6 \pm 0.4$ GeV.

In Fig. 1, the width of the bands is determined by the parametric uncertainty induced by the uncertainty in the top quark mass and α_s. It is dominated by the uncertainty in the top mass. The top mass has been constrained by kinematic fits in combined analyses of Tevatron [25] and LHC [26] data, and may also be stringently constrained in the future by cross section measurements (see, e.g., Ref. [27]). For now, we consider the range $m_t^{\text{pole}} = 173.3 \pm 1.8$ GeV. The resulting parametric uncertainty is from 0.5–2 GeV; it exceeds the experimental uncertainty and is comparable to that expected from four- and higher-loop effects in the theoretical prediction.

In Fig. 2, we compare our results to those of two-loop codes. The two-loop results differ significantly from each other, with differences of up to 4 GeV for top squark masses in the 1–10 TeV range shown. The three-loop results are within this range for ~TeV top squark masses, as found in Refs. [5,6]. However, for multi-TeV top squark masses, the three-loop contributions may significantly enhance m_h.

Some of the differences between the two-loop results can be explained by different default choices for the renormalization scale. They also differ in how the running top mass is extracted from its pole mass. This difference is formally of higher order [28]. The different treatment of parameters also explains the difference between H3M's two-loop results

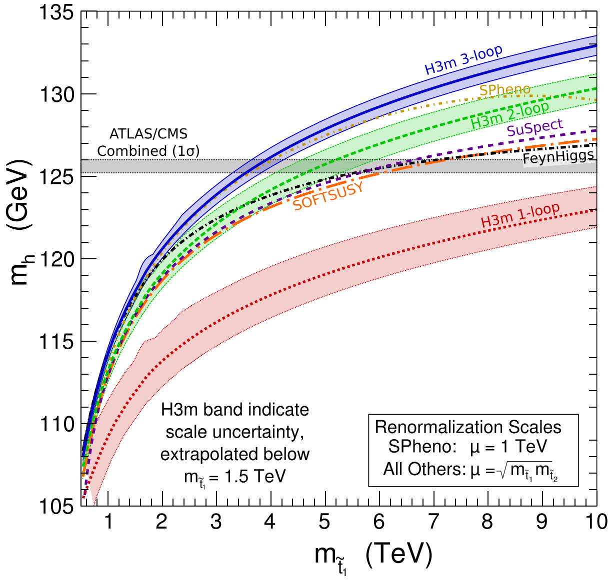

FIG. 2 (color online). Comparison of H3M results with the two-loop results of FEYNHIGGS [10–13], SOFTSUSY [14], SUSPECT [15], and SPHENO [16,17]. The H3M bands indicate the uncertainty from varying the renormalization scale between $M_S/2$ and $2M_S$. The supersymmetry parameters are as in Fig. 1.

and FEYNHIGGS. For example, FEYNHIGGS uses one-loop running for α_s and m_t, which is formally correct since the two-loop results are leading order in α_s.

Results for mSUGRA and implications for supersymmetry at the LHC.—To determine the implications of the three-loop corrections for the LHC, we consider here the well-known framework of minimal supergravity (mSUGRA), defined in terms of grand unified theory–scale parameters, for which detailed collider studies have been carried out.

In Fig. 3, we show contours of m_h with three-loop corrections in two well-studied (m_0, $M_{1/2}$) planes of mSUGRA. To highlight the regions of parameter space preferred by m_h, at each point in parameter space, we define a theoretical uncertainty $\Delta_{\text{th}} \equiv \sqrt{(\Delta_{\text{pert}})^2 + (\Delta_{\text{para}})^2}$, where

$$\Delta_{\text{pert}} \equiv \frac{1}{2}\left| m_h^{(\text{three-loop})} - m_h^{(\text{two-loop})} \right|,$$

$$\Delta_{\text{para}} \equiv \left| m_h\begin{pmatrix} m_t = 175.1\ \text{GeV} \\ \alpha_s = 0.1177 \end{pmatrix} - m_h\begin{pmatrix} m_t = 173.3\ \text{GeV} \\ \alpha_s = 0.1184 \end{pmatrix} \right|. \quad (4)$$

The quantity Δ_{pert} is the estimated uncertainty from neglecting higher-order terms in the perturbation series. It is motivated by observing that the scale variation of the two-loop prediction underestimates the three-loop corrections, and is typically in the 0.5–1.5 GeV range. The parametric uncertainty Δ_{para} arises dominantly from the uncertainty in the top quark mass. In the figure, we shade

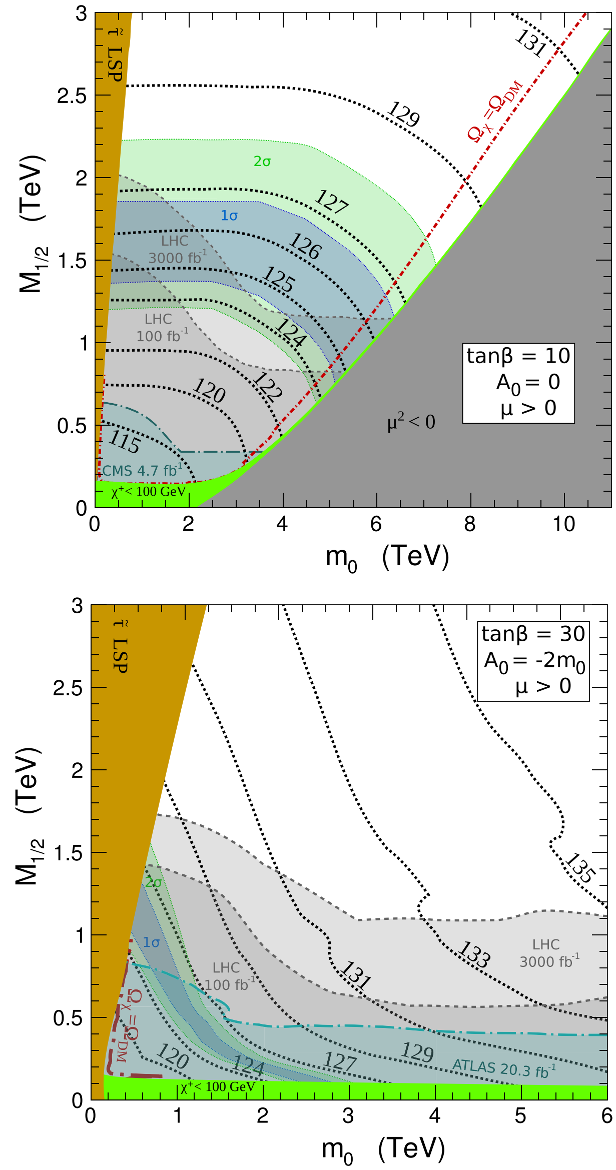

FIG. 3 (color online). Three-loop H3M m_h contours in two (m_0, $M_{1/2}$) planes of mSUGRA, with $\tan\beta$, A_0, and sign(μ) as indicated. In the dark blue (light green) shaded regions, the theoretical prediction is within Δ_{th} ($2\Delta_{\text{th}}$) of the experimental central value. On the $\Omega_\chi = \Omega_{\text{DM}}$ contour, thermal relic neutralinos are all the dark matter. Top: Negligible top squark mixing, with current exclusion contour from CMS [31], and projected sensitivities of the 14 TeV LHC and its high-luminosity upgrade [32]. Bottom: Significant top squark mixing, with current exclusion contour from ATLAS [33], and projected sensitivities of the 14 TeV LHC and its high-luminosity upgrade [34].

regions where the calculated m_h is within Δ_{th} and $2\Delta_{\text{th}}$ of the experimental central value 125.6 GeV.

The positive three-loop terms significantly impact the preferred range of superpartner masses and the prospects for supersymmetry discovery at the LHC. In the top panel of Fig. 3, $A_0 = 0$ and top squark mixing is negligible throughout the plane. Requiring that the theoretical prediction be within $2\Delta_{\rm th}$ of the experimental central value, and imposing the further requirement that thermal relic neutralinos make up all the dark matter (the focus point region [29,30]), scalar mass parameters as low as $m_0 \sim$ 4–5 TeV, corresponding to top squark masses as low as 3–4 TeV, and gluino masses as low as $m_{\tilde{g}} \simeq 2.8 M_{1/2} \approx$ 2 TeV are consistent with the measured Higgs mass. These are far lighter than the squark masses required if only one- and two-loop corrections to m_h are included. Current bounds do not challenge this parameter space [31], but the 14 TeV LHC with 100 fb^{-1} will already start probing the favored parameter space, and a high-luminosity upgrade to 3 ab^{-1} may probe most of it [32]. The LHC reach was extrapolated from a study that used $\tan\beta = 45$ [32] by transferring the $(m_{\tilde{q}}, m_{\tilde{g}})$ values on the reach contours to the space with $\tan\beta = 10$. The sensitivities are determined by searches for multiple jets and missing energy along with a variable number of leptons and are expected to be approximately independent of $\tan\beta$. Of course, lighter squark masses and brighter discovery prospects are possible if one relaxes the cosmological requirement.

If there is significant top squark mixing, the implications may be even more dramatic. This is illustrated in the bottom panel, of Fig. 3, where $A_0 = -2m_0$. With the three-loop corrections included, the preferred region moves to m_0 as low as 1 TeV, and the 2σ region even overlaps the region with the correct thermal relic density of neutralinos (the stau coannihilation region). Current bounds [33] exclude some of the favored region, but the 14 TeV LHC with 100 fb^{-1} will probe most of it, and it will be explored fully by the LHC high-luminosity upgrade [34].

Conclusions.—Three-loop contributions to the Higgs boson mass may be as large as 3 GeV in supersymmetric theories with multi-TeV superpartners. Given the extreme sensitivity of the top squark mass to such changes, this lowers the preferred top squark mass to as low as 3–4 TeV, with striking implications for supersymmetry discovery at the LHC. In models with a characteristic squark mass scale, these results imply that even without significant mixing or additional particles, first and second generation squarks may be within reach of the 14 TeV LHC with 100 fb^{-1}, with much more promising prospects for a high-luminosity upgrade. Given the rapidly diminishing experimental uncertainty on m_h, these results highlight the importance of improved theoretical calculations of m_h, incorporating improved determinations of the top quark mass, to refine the implications of the Higgs boson discovery for supersymmetry.

We thank S. Heinemeyer, S. P. Martin, L. Mihaila, W. Porod, P. Slavich, M. Steinhauser, X. Tata, and N. Zerf for useful discussions, and P. Draper for collaboration in early stages of this work. J. L. F. is supported in part by U.S. NSF Grant No. PHY-0970173 and by the Simons Foundation. P. K. is supported by the DFG through Grant No. SFB/TR-9 and by the Helmholtz Alliance "Physics at the Terascale." S. P. is supported in part by U.S. DOE Grant No. DE-FG02-04ER41268. D. S. is supported in part by U.S. DOE Grant No. DE-FG02-92ER40701 and by the Gordon and Betty Moore Foundation through Grant No. 776 to the Caltech Moore Center for Theoretical Cosmology and Physics.

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